Research trends in Combinatorial Optimization

P.C. Pop

Department of Mathematics and Computer Science
Technical University of Cluj-Napoca, North Univ. Center of Baia Mare

5th March 2013

Outline

Introduction

- Technical University of Cluj-Napoca
- Faculty of Sciences, North University Center of Baia Mare
- International Conference on Applied Mathematics (25-28 September 2013)
- Carpathian Journal of Mathematics

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic!
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
- The combinatorial optimization problems may arise in a wide variety of important fields such as transportation, computer networking, telecommunications, location, planning, distribution problems, etc.
- Important and significant results have been obtained on the theory, algorithms and applications in the last few decades.

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic!
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
- The combinatorial optimization problems may arise in a wide variety of important fields such as transportation, computer networking, telecommunications, location, planning, distribution problems, etc.
- Important and significant results have been obtained on the theory, algorithms and applications in the last few decades.

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic!
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
- The combinatorial optimization problems may arise in a wide variety of important fields such as transportation, computer networking, telecommunications, location, planning, distribution problems, etc.
- Important and significant results have been obtained on the theory, algorithms and applications in the last few decades.

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic!
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
- The combinatorial optimization problems may arise in a wide variety of important fields such as transportation, computer networking, telecommunications, location, planning, distribution problems, etc.
- Important and significant results have been obtained on the theory, algorithms and applications in the last few decades.

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic!
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
- The combinatorial optimization problems may arise in a wide variety of important fields such as transportation, computer networking, telecommunications, location, planning, distribution problems, etc.
- Important and significant results have been obtained on the theory, algorithms and applications in the last few decades.

Combinatorial Optimization

- Combinatorial optimization is a fascinating topic!
- Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.
- It is a branch of applied mathematics and computer science, related to operations research, algorithm theory and computational complexity theory that sits at the intersection of several fields, including artificial intelligence, mathematics and software engineering.
- The combinatorial optimization problems may arise in a wide variety of important fields such as transportation, computer networking, telecommunications, location, planning, distribution problems, etc.
- Important and significant results have been obtained on the theory, algorithms and applications in the last few decades.

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - complete$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - \textit{complete}$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - complete$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - complete$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - \mathit{complete}$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - complete$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP}-$ *complete*.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- The first step in studying a combinatorial optimization problem is to find out whether the problem is "easy" or "hard".
- The combinatorial optimization problems are partitioned into two sets: $\mathcal{P}$ and $\mathcal{NP} - complete$.
- A problem in the set $\mathcal{P}$ can be solved by an algorithm whose solution time grows as a polynomial function of the size of the problem.
- Examples:
  - shortest path problem
  - assignment problems
  - minimum spanning tree problem
  - linear programming problems

Complexity aspects

- For the problems in the set $\mathcal{NP} - complete$, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP}$ − *complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP} -$ *complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP}-$ *complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP} - \textit{complete}$, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set *$\mathcal{NP}-$complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP} -$ *complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP} -$ *complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Complexity aspects

- For the problems in the set $\mathcal{NP} -$ *complete*, no polynomial algorithms have been found.
- Examples:
  - traveling salesman problem
  - vehicle routing problem
  - number partitioning problem
  - graph coloring problem
  - set cover problem
  - scheduling problems
  - green logistics
  - ...

Modeling combinatorial optimization problems

- Combinatorial optimization models are often referred to as integer programming models where some or all of the variables can take on only a finite number of alternative possibilities.

$$(IP) \quad \begin{array}{lrcl} \min & c^T x & & \\ s.t. & Ax & \leq & b \\ & x & \in & \mathbb{Z}^n \end{array}$$

- If we allow some variables x_i to be continuous instead of integer, i.e., $x_i \in \mathbb{R}$ instead of $x_i \in \mathbb{Z}$, then we obtain a *mixed integer programming problem*, denoted by (MIP).
- In comparison to linear programming problems, solving integer programming problems is a difficult task.

Modeling combinatorial optimization problems

- Combinatorial optimization models are often referred to as integer programming models where some or all of the variables can take on only a finite number of alternative possibilities.

$$(IP)\quad \begin{array}{lrcl} \min & c^T x & & \\ s.t. & Ax & \leq & b \\ & x & \in & \mathbb{Z}^n \end{array}$$

- If we allow some variables x_i to be continuous instead of integer, i.e., $x_i \in \mathbb{R}$ instead of $x_i \in \mathbb{Z}$, then we obtain a *mixed integer programming problem*, denoted by (MIP).
- In comparison to linear programming problems, solving integer programming problems is a difficult task.

Modeling combinatorial optimization problems

- Combinatorial optimization models are often referred to as integer programming models where some or all of the variables can take on only a finite number of alternative possibilities.

$$
(IP) \quad \begin{array}{lrcl} \min & c^T x & & \\ s.t. & Ax & \leq & b \\ & x & \in & \mathbb{Z}^n \end{array}
$$

- If we allow some variables x_i to be continuous instead of integer, i.e., $x_i \in \mathbb{R}$ instead of $x_i \in \mathbb{Z}$, then we obtain a *mixed integer programming problem*, denoted by (MIP).
- In comparison to linear programming problems, solving integer programming problems is a difficult task.

Solving combinatorial optimization problems

- Finding good solutions for hard minimization problems in combinatorial optimization requires the consideration of two issues:
  - calculation of an upper bound that is as close as possible to the optimum;
  - calculation of a lower bound that is as close as possible to the optimum.

  When optimizing combinatorial problems, there is always a trade-off between the computational effort (and hence the running time) and the quality of the obtained solution.
- We may either try to solve the problem to optimality with an exact algorithm, or choose for an approximation or heuristic algorithm, which uses less running time but does not guarantee optimality of the solution.

Solving combinatorial optimization problems

- Finding good solutions for hard minimization problems in combinatorial optimization requires the consideration of two issues:
  - calculation of an upper bound that is as close as possible to the optimum;
  - calculation of a lower bound that is as close as possible to the optimum.

  When optimizing combinatorial problems, there is always a trade-off between the computational effort (and hence the running time) and the quality of the obtained solution.
- We may either try to solve the problem to optimality with an exact algorithm, or choose for an approximation or heuristic algorithm, which uses less running time but does not guarantee optimality of the solution.

Solving combinatorial optimization problems

- Finding good solutions for hard minimization problems in combinatorial optimization requires the consideration of two issues:
  - calculation of an upper bound that is as close as possible to the optimum;
  - calculation of a lower bound that is as close as possible to the optimum.

  When optimizing combinatorial problems, there is always a trade-off between the computational effort (and hence the running time) and the quality of the obtained solution.
- We may either try to solve the problem to optimality with an exact algorithm, or choose for an approximation or heuristic algorithm, which uses less running time but does not guarantee optimality of the solution.

Solving combinatorial optimization problems

- Finding good solutions for hard minimization problems in combinatorial optimization requires the consideration of two issues:
  - calculation of an upper bound that is as close as possible to the optimum;
  - calculation of a lower bound that is as close as possible to the optimum.

  When optimizing combinatorial problems, there is always a trade-off between the computational effort (and hence the running time) and the quality of the obtained solution.
- We may either try to solve the problem to optimality with an exact algorithm, or choose for an approximation or heuristic algorithm, which uses less running time but does not guarantee optimality of the solution.

Solving combinatorial optimization problems

- **Exact algorithms** guarantee to find the optimal solution, but may take an exponential number of iterations. They are in practice typically applicable to small instances only, due to high running times caused by the high complexity and include:
  - branch-and-bound
  - branch-and-cut
  - cutting plane
  - dynamic programming.
- **Approximation algorithms** that provide in polynomial time a sub-optimal solution together with a bound on the degree of suboptimality.
  - Christofides approximation algorithm: the cost of the solution produced by the algorithm is within 3/2 of the optimum.
- **Heuristic algorithms** that provide a sub-optimal solution, but with no guarantee on its quality. These algorithms are especially addressed for large instances of the optimization problems.
  - classical heuristics (standard construction and improvement procedures)
  - metaheuristic algorithms.

Solving combinatorial optimization problems

- **Exact algorithms** guarantee to find the optimal solution, but may take an exponential number of iterations. They are in practice typically applicable to small instances only, due to high running times caused by the high complexity and include:
  - branch-and-bound
  - branch-and-cut
  - cutting plane
  - dynamic programming.
- **Approximation algorithms** that provide in polynomial time a sub-optimal solution together with a bound on the degree of suboptimality.
  - Christofides approximation algorithm: the cost of the solution produced by the algorithm is within 3/2 of the optimum.
- **Heuristic algorithms** that provide a sub-optimal solution, but with no guarantee on its quality. These algorithms are especially addressed for large instances of the optimization problems.
  - classical heuristics (standard construction and improvement procedures)
  - metaheuristic algorithms.

Solving combinatorial optimization problems

- **Exact algorithms** guarantee to find the optimal solution, but may take an exponential number of iterations. They are in practice typically applicable to small instances only, due to high running times caused by the high complexity and include:
  - branch-and-bound
  - branch-and-cut
  - cutting plane
  - dynamic programming.
- **Approximation algorithms** that provide in polynomial time a sub-optimal solution together with a bound on the degree of suboptimality.
  - Christofides approximation algorithm: the cost of the solution produced by the algorithm is within 3/2 of the optimum.
- **Heuristic algorithms** that provide a sub-optimal solution, but with no guarantee on its quality. These algorithms are especially addressed for large instances of the optimization problems.
  - classical heuristics (standard construction and improvement procedures)
  - metaheuristic algorithms.

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

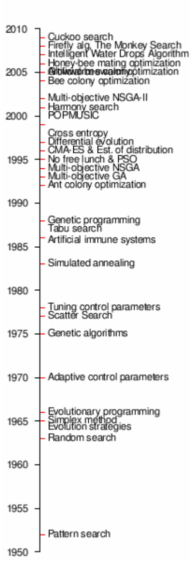
Metaheuristics for solving combinatorial optimization problems

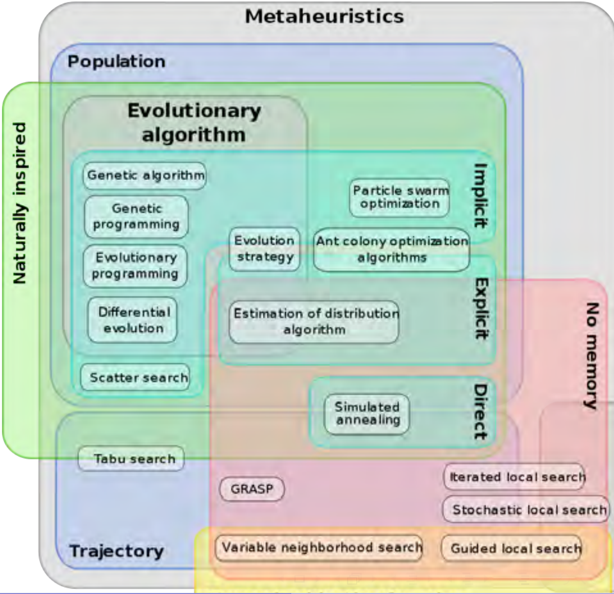
- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems

- "A metaheuristic is a set of concepts that can be used to define heuristic methods that can be applied to a wide set of different problems. In other words, a metaheuristic can be seen as a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem." (Metaheuristics Network Website 2000).
- Popular metaheuristics for combinatorial problems include:
  - simulated annealing by Kirkpatrick et al. (1983)
  - genetic algorithms by Holland et al. (1975)
  - ant colony optimization by Dorigo (1992)
  - scatter search (1977);
  - tabu search (1986) by Glover;
  - Variable Neighborhood Search (1997) by Mladenovic and Hansen;
  - ...

Metaheuristics for solving combinatorial optimization problems





Intelligent methods for combinatorial optimization problems

Hybrid algorithms exploit the good properties of different methods by applying them to problems they can efficiently solve.

We plan to investigate more sophisticated hybrid approaches based on combination of:

- exact methods based on integer linear programming with local search based metaheuristics.
- local search methods within genetic search in order to achieve a balance between exploration and exploitation.

Intelligent methods for combinatorial optimization problems

Hybrid algorithms exploit the good properties of different methods by applying them to problems they can efficiently solve.

We plan to investigate more sophisticated hybrid approaches based on combination of:

- exact methods based on integer linear programming with local search based metaheuristics.
- local search methods within genetic search in order to achieve a balance between exploration and exploitation.

The Traveling Salesman Problem (TSP)

- Mathematical problems related to the traveling salesman problem were treated in the 1800s by the Irish mathematician Sir W.R. Hamilton and by the British mathematician T.P Kirkman.
- We are given a set of cities and a cost associated with traveling between each pair of cities and we want to find the least cost route traveling through every city and ending up back at the starting city.
- Applications of the TSP: Drilling Circuit Boards, Gene Sequencing, Emergency Management, Planning, Logistics, etc.
- The TSP is a combinatorial problem defined on a graph $G = (V; E)$
  - V is the set of customers.
  - E is the set of travel links between the customers.

The Traveling Salesman Problem (TSP)

- Mathematical problems related to the traveling salesman problem were treated in the 1800s by the Irish mathematician Sir W.R. Hamilton and by the British mathematician T.P Kirkman.
- We are given a set of cities and a cost associated with traveling between each pair of cities and we want to find the least cost route traveling through every city and ending up back at the starting city.
- Applications of the TSP: Drilling Circuit Boards, Gene Sequencing, Emergency Management, Planning, Logistics, etc.
- The TSP is a combinatorial problem defined on a graph $G = (V; E)$
  - V is the set of customers.
  - E is the set of travel links between the customers.

The Traveling Salesman Problem (TSP)

- Mathematical problems related to the traveling salesman problem were treated in the 1800s by the Irish mathematician Sir W.R. Hamilton and by the British mathematician T.P Kirkman.
- We are given a set of cities and a cost associated with traveling between each pair of cities and we want to find the least cost route traveling through every city and ending up back at the starting city.
- Applications of the TSP: Drilling Circuit Boards, Gene Sequencing, Emergency Management, Planning, Logistics, etc.
- The TSP is a combinatorial problem defined on a graph $G = (V; E)$
  - V is the set of customers.
  - E is the set of travel links between the customers.

The Traveling Salesman Problem (TSP)

- Mathematical problems related to the traveling salesman problem were treated in the 1800s by the Irish mathematician Sir W.R. Hamilton and by the British mathematician T.P Kirkman.
- We are given a set of cities and a cost associated with traveling between each pair of cities and we want to find the least cost route traveling through every city and ending up back at the starting city.
- Applications of the TSP: Drilling Circuit Boards, Gene Sequencing, Emergency Management, Planning, Logistics, etc.
- The TSP is a combinatorial problem defined on a graph $G = (V; E)$
  - V is the set of customers.
  - E is the set of travel links between the customers.

The Traveling Salesman Problem (TSP)

- Mathematical problems related to the traveling salesman problem were treated in the 1800s by the Irish mathematician Sir W.R. Hamilton and by the British mathematician T.P Kirkman.
- We are given a set of cities and a cost associated with traveling between each pair of cities and we want to find the least cost route traveling through every city and ending up back at the starting city.
- Applications of the TSP: Drilling Circuit Boards, Gene Sequencing, Emergency Management, Planning, Logistics, etc.
- The TSP is a combinatorial problem defined on a graph $G = (V; E)$
  - V is the set of customers.
  - E is the set of travel links between the customers.

The Traveling Salesman Problem (TSP)

- Mathematical problems related to the traveling salesman problem were treated in the 1800s by the Irish mathematician Sir W.R. Hamilton and by the British mathematician T.P Kirkman.
- We are given a set of cities and a cost associated with traveling between each pair of cities and we want to find the least cost route traveling through every city and ending up back at the starting city.
- Applications of the TSP: Drilling Circuit Boards, Gene Sequencing, Emergency Management, Planning, Logistics, etc.
- The TSP is a combinatorial problem defined on a graph $G = (V; E)$
  - V is the set of customers.
  - E is the set of travel links between the customers.

How difficult is to solve the Traveling Salesman Problem?

In the picture is represented the optimal Hamiltonian tour visiting the first 15 biggest cities from Germany. The total number of Hamiltonian tours is $15! = 43\,589\,145\,600$.

A mathematical model for TSP

$$\min \sum_{e=(i,j)\in E} c_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, ..., n$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, ..., n$$

$$\sum_{(i,j)\in E(S)} \quad \leq |S| - 1, \forall S \subset V, \ |S| > 1, \ S \neq V$$

$$x_{ij} \in \{0, 1\}, \forall e = (i,j) \in E.$$

where $x_{ij} = 1$ if the edge $e = (i,j)$ is selected in the tour and 0 otherwise.

Generalized network design problems

- Many network design problems can be generalized in a natural way by considering a related problem on a clustered graph, where the original problem's feasibility constraints are expressed in terms of the clusters, i.e. node sets instead of individual nodes.
- Given an undirected weighted graph $G = (V, E)$ with node set V and edge set E. The nodes are partitioned into a given number of node sets called *clusters* and edges are defined between any two nodes belonging to different clusters and to each edge $e \in E$ we associate a nonnegative cost c_e.
- The goal of these problems is to find a subgraph $F = (S, T)$ of G where the subset of nodes $S = \{v_1, ..., v_m\} \subset V$ is containing exactly one node from each cluster with different requirements to be fulfilled by the subset of edges $T \subset E$ depending on the actual optimization problem.

Generalized network design problems

- Many network design problems can be generalized in a natural way by considering a related problem on a clustered graph, where the original problem's feasibility constraints are expressed in terms of the clusters, i.e. node sets instead of individual nodes.
- Given an undirected weighted graph $G = (V, E)$ with node set V and edge set E. The nodes are partitioned into a given number of node sets called *clusters* and edges are defined between any two nodes belonging to different clusters and to each edge $e \in E$ we associate a nonnegative cost c_e.
- The goal of these problems is to find a subgraph $F = (S, T)$ of G where the subset of nodes $S = \{v_1, ..., v_m\} \subset V$ is containing exactly one node from each cluster with different requirements to be fulfilled by the subset of edges $T \subset E$ depending on the actual optimization problem.

Generalized network design problems

- Many network design problems can be generalized in a natural way by considering a related problem on a clustered graph, where the original problem's feasibility constraints are expressed in terms of the clusters, i.e. node sets instead of individual nodes.
- Given an undirected weighted graph $G = (V, E)$ with node set V and edge set E. The nodes are partitioned into a given number of node sets called *clusters* and edges are defined between any two nodes belonging to different clusters and to each edge $e \in E$ we associate a nonnegative cost c_e.
- The goal of these problems is to find a subgraph $F = (S, T)$ of G where the subset of nodes $S = \{v_1, ..., v_m\} \subset V$ is containing exactly one node from each cluster with different requirements to be fulfilled by the subset of edges $T \subset E$ depending on the actual optimization problem.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

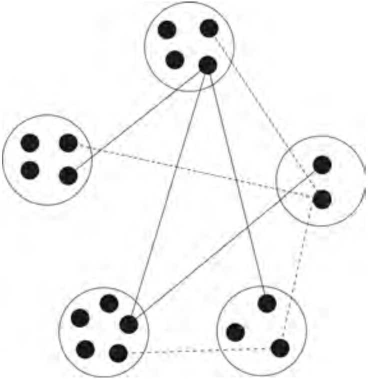
Generalized network design problems

- In this way, it is introduced the class of generalized network design problems (generalized combinatorial optimization problems, selective combinatorial optimization problems):
  - the generalized minimum spanning tree problem
  - the generalized traveling salesman problem
  - the generalized vehicle routing problem
  - the generalized fixed-charge network design problem
  - the generalized minimum edge-biconnected network problem
  - the selective graph coloring problem
  - ...

P.C. Pop, Generalized Network Design Problems, Modeling and Optimization, De Gruyter, Germany, ISBN 978-3-11-026768-6, 2012.

The Generalized Minimum Spanning Tree Problem

- The GMSTP was introduced by Myung et al. (1995).
- The GMSTP asks for finding a minimum-cost tree T spanning a subset of nodes which includes exactly one node from each cluster V_i, $i \in \{1, ..., m\}$.



The Generalized Minimum Spanning Tree Problem

- Applications: design of metropolitan and regional area networks, determining the locations of regional service centers, energy transportation, agricultural irrigation, etc.
- Achieved results:

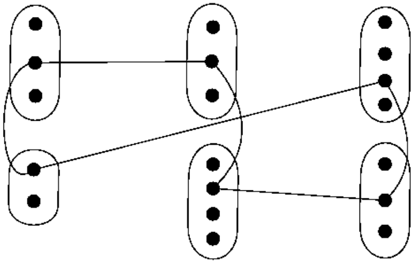
P.C. Pop, W. Kern and G. Still, A New Relaxation Method for the Generalized Minimum Spanning Tree Problem, European Journal of Operational Research, Vol. 170, pp. 900-908, 2006.

P.C. Pop, On the Prize-Collecting Generalized Minimum Spanning Tree Problem, Annals of Operations Research, Vol. 150, No. 1, pp. 193-204, 2007.

P.C. Pop, A survey of different integer programming formulations of the generalized minimum spanning tree problem, Carpathian Journal of Mathematics, Vol. 25, No. 1, pp. 104-118, 2009.

The Generalized Traveling Salesman Problem (GTSP)

- The GTSP was introduced independently by Henry-Labordere (1969), Srivastava et al. (1969) and Saskena (1970).
- The goal of the GTSP is to find a minimum-cost tour *H* spanning a subset of nodes such that *H* contains exactly one node from each cluster V_i, $i \in \{1, ..., m\}$. We will call such a cycle a generalized Hamiltonian tour.

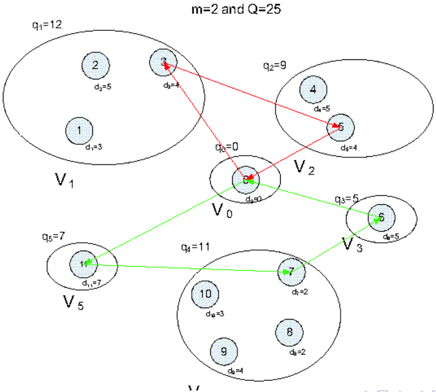


The Generalized Traveling Salesman Problem (GTSP)

- Applications: location and logistics problems, routing problems, telecommunication, manufacture of microchips, planning, railway optimization, etc.
- Achieved results:
  - P.C. Pop and S. Iordache, A Hybrid Heuristic Approach for Solving the Generalized Traveling Saleasman Problem, in Proc. of GECCO 2011, Association for Computing Machinery, pp. 481-488, 2011, ISBN: 978-1-4503-0557-0.
  - P.C. Pop, O. Matei and C. Sabo, A New Approach for Solving the Generalized Traveling Salesman Problem, in Proc. of HM 2010, Editors M.J. Blesa et al., Lecture Notes in Computer Science, Springer, Vol. 6373, pp. 62-72, 2010.
  - O. Matei and P.C. Pop, An efficient genetic algorithm for solving the generalized traveling salesman problem, in Proc. of 6-th IEEE International Conference on Intelligent Computer Communication and Processing, pp. 87-92, 2010.

The Generalized Vehicle Routing Problem (GVRP)

- The GVRP was introduced by Ghiani and Improta 2000.
- The goal of the problem is to design the optimal delivery or collection routes, subject to capacity restrictions, from a given depot to a number of predefined, mutually exclusive and exhaustive node-sets (clusters).



The Generalized Vehicle Routing Problem (GVRP)

- Applications: post-box collection problem, design of configurations for automated guided vehicles, distribution problems, etc.
- Achieved results:

P.C. Pop, O. Matei and C. Pop Sitar, An Improved Hybrid Algorithm for Solving the Generalized Vehicle Routing Problem, Neurocomputing (to appear), DOI information: 10.1016/j.neucom.2012.03.032.

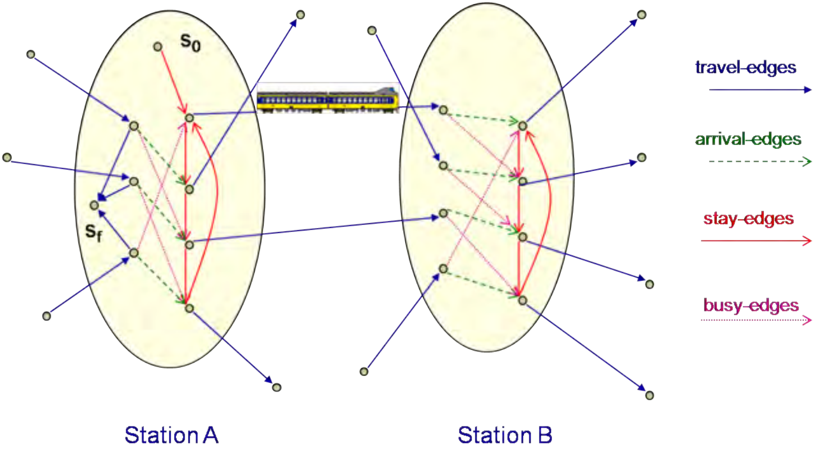
P.C. Pop, I. Kara and A. Horvat Marc, New Mathematical Models of the Generalized Vehicle Routing Problem and Extensions, Applied Mathematical Modelling, Elsevier, Vol. 36, Issue 1, pp. 97-107, 2012.

P.C. Pop, C. Pop Sitar, I. Zelina, V. Lupse and C. Chira, Heuristic algorithms for solving the generalized vehicle routing problem, International Journal of Computers, Communications & Control, Vol. 6, No. 1, pp. 158-166, 2011.

The Railway Traveling Salesman Problem

- We assume that we are given a set of stations, a timetable regarding trains connecting these stations, an initial station, a subset B of the stations and a starting time.
- A salesman wants to travel from the initial station, starting not earlier than the designated time, to every station in B and finally returns back to the initial station, subject to the constrained that he/she spends the necessary amount of time in each station of B to carry out his/her business.
- The goal is to find a set of train connections such that the overall time of the journey is minimized. This problem is called the *Railway Traveling Salesman Problem*.
- The RTSP is NP-hard and it is related to the Generalized Traveling Salesman Problem.

The Railway Traveling Salesman Problem (RTSP)



G. Hadjicharalambous, **P.C. Pop**, E. Pyrga, G. Tsaggouris and C.D. Zaroliagis, The Railway Travelling Salesman Problem, in Algorithmic Methods for Railway Optimization, Lecture Notes in Computer Science, Vol. 4359, pp. 264-275, 2007.

The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

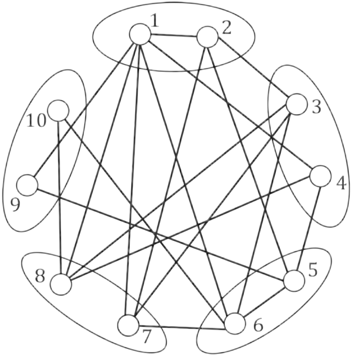
The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

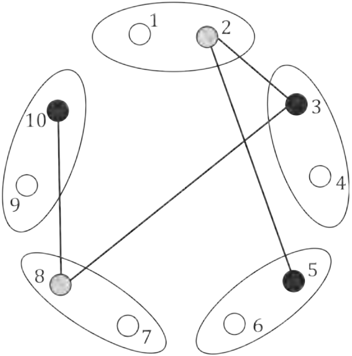
The selective graph coloring problem

- Consider an undirected graph $G = (V, E)$ and a partition $V_1, ..., V_p$ of its vertex set V.
- For some integer $k \geq 1$, the selective graph coloring problem consists in finding a subset $V^* \subseteq V$ such that:
  - $|V^* \cap V_i| = 1$ for all $i \in \{1, ..., p\}$
  - the graph induced by V^* is k-colorable.
- Such a coloring will be called a selective graph coloring.
- Since the classical graph coloring problem is a special case of the selective graph coloring problem, it follows that the selective graph coloring problem is NP-hard in general.
- Applications: scheduling problems.

The selective graph coloring problem



a)



b)

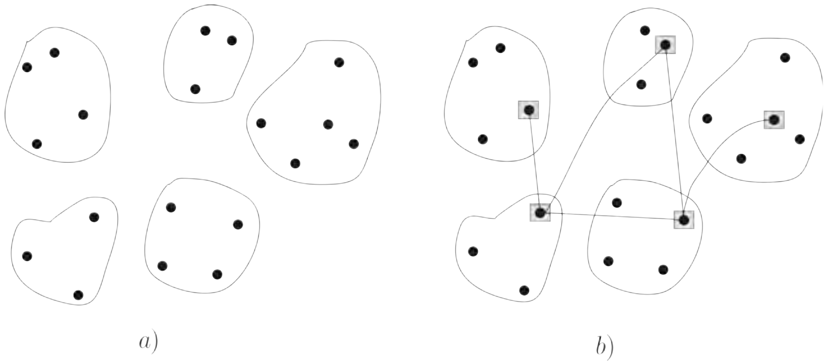
The selective graph coloring problem

- Achieved results:
  - M. Demange, J. Monnot, P.C. Pop and B. Reis, On the complexity of the selective graph coloring problem in some special classes of graphs, Theoretical Computer Science (submitted).
  - M. Demange, J. Monnot, P.C. Pop and B. Reis, Selective Graph Coloring in Some Special Classes of Graphs, in Proc. of ISCO 2012, Lecture Notes in Computer Science, Vol. 7422, pp. 320-331, Springer, 2012.
  - P.C. Pop, B. Hu and G. Raidl, Metaheuristic approaches for the Partition Graph Coloring Problem, in Proc. of EUROCAST 2013, to appear in Lecture Notes in Computer Science, Springer, 2013.

The Generalized Fixed-Charge Network Design Problem

- The Generalized Fixed-Charge Network Design Problem (GFCNDP) was introduced by Thomadsen and Stidsen (2007).
- The GFCNDP asks for finding the cheapest backbone network connecting exactly one hub (node) from each of the given clusters.
- It is an important subproblem when constructing hierarchical telecommunication networks.
- An illustration of the GFCNDP presenting the clusters and an example of backbone network is shown in the following figure.

The Generalized Fixed-Charge Network Design Problem (GFCNDP)



Conclusions

- Progress in combinatorial optimization relies on an appropriate modelling of problems and the study of the combinatorial structure of these problems.
- Novel research contributions are expected on exact and heuristic algorithms and, in particular, the combination of these techniques.
- It is widely acknowledged that breakthroughs in the domain of combinatorial optimization will most likely come from an integration of both exact and heuristic methods to exploit the complementarity of these two types of approaches.
- Implementation of solution methods for large-scale, practically relevant problems. Applications drive research and often require solving ever larger and more realistic models of real-world optimization problems.
- dynamic combinatorial optimization problems.

Conclusions

- Progress in combinatorial optimization relies on an appropriate modelling of problems and the study of the combinatorial structure of these problems.
- Novel research contributions are expected on exact and heuristic algorithms and, in particular, the combination of these techniques.
- It is widely acknowledged that breakthroughs in the domain of combinatorial optimization will most likely come from an integration of both exact and heuristic methods to exploit the complementarity of these two types of approaches.
- Implementation of solution methods for large-scale, practically relevant problems. Applications drive research and often require solving ever larger and more realistic models of real-world optimization problems.
- dynamic combinatorial optimization problems.

Conclusions

- Progress in combinatorial optimization relies on an appropriate modelling of problems and the study of the combinatorial structure of these problems.
- Novel research contributions are expected on exact and heuristic algorithms and, in particular, the combination of these techniques.
- It is widely acknowledged that breakthroughs in the domain of combinatorial optimization will most likely come from an integration of both exact and heuristic methods to exploit the complementarity of these two types of approaches.
- Implementation of solution methods for large-scale, practically relevant problems. Applications drive research and often require solving ever larger and more realistic models of real-world optimization problems.
- dynamic combinatorial optimization problems.

Conclusions

- Progress in combinatorial optimization relies on an appropriate modelling of problems and the study of the combinatorial structure of these problems.
- Novel research contributions are expected on exact and heuristic algorithms and, in particular, the combination of these techniques.
- It is widely acknowledged that breakthroughs in the domain of combinatorial optimization will most likely come from an integration of both exact and heuristic methods to exploit the complementarity of these two types of approaches.
- Implementation of solution methods for large-scale, practically relevant problems. Applications drive research and often require solving ever larger and more realistic models of real-world optimization problems.
- dynamic combinatorial optimization problems.

Conclusions

- Progress in combinatorial optimization relies on an appropriate modelling of problems and the study of the combinatorial structure of these problems.
- Novel research contributions are expected on exact and heuristic algorithms and, in particular, the combination of these techniques.
- It is widely acknowledged that breakthroughs in the domain of combinatorial optimization will most likely come from an integration of both exact and heuristic methods to exploit the complementarity of these two types of approaches.
- Implementation of solution methods for large-scale, practically relevant problems. Applications drive research and often require solving ever larger and more realistic models of real-world optimization problems.
- dynamic combinatorial optimization problems.